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Inhomogeneous Semiconductors

The Semiclassical Treatment of
Inhomogeneous Solids

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the Nonequilibrium p - n Junction

As used by music lovers, the term “solid state physics” refers only to the subject of inhomogeneous semiconductors, and it would be more accurate if it were this latter phrase that festooned the brows of countless tuners and amplifiers. The prevailing usage reflects the fact that modern solid state physics has had its most dramatic and extensive technological consequences through the electronic properties of semiconducting devices. These devices use semiconducting crystals in which the concentrations of donor and acceptor impurities have been made nonuniform in a carefully controlled manner. We shall not attempt to survey here the great variety of semiconducting devices, but will only describe the broad physical principles that underlie their operation. These principles come into play in determining how the densities and currents of electrons and holes are distributed in an inhomogeneous semiconductor, both in the absence and in the presence of an applied electrostatic potential.

The inhomogeneous semiconductors of interest are, ideally, single crystals in which the local concentration of donor and acceptor impurities varies with position. One way to make such crystals is to vary the concentration of impurities in the “melt” as the growing crystal is slowly extracted, thus producing a variation in impurity concentration along one spatial direction. Delicate methods of fabrication are needed because it is generally important, for efficient operation, that there be no great increase in electronic scattering associated with the variation in impurity concentration.

We shall illustrate the physics of inhomogeneous semiconductors by considering the simplest example, the p - n junction. This is a semiconducting crystal in which the impurity concentration varies only along a given direction (taken to be the x -axis) and only in a small region (taken to be around $x = 0$). For negative x the crystal has a preponderance of acceptor impurities (i.e., it is p -type) while for positive x it has a preponderance of donor impurities (i.e., it is n -type) (Figure 29.1). The manner in which the densities of donors and acceptors $N_d(x)$ and $N_a(x)$ vary with position is

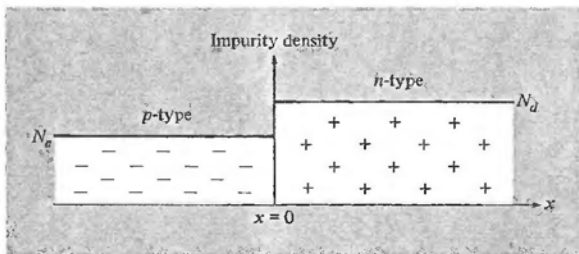


Figure 29.1

The impurity densities along a p - n junction in the case of an “abrupt junction,” for which donor impurities dominate at positive x , and acceptor impurities at negative x . The donors are represented by (+) to indicate their charge when ionized, and the acceptors by (-). For a junction to be abrupt, the region about $x = 0$ where the impurity concentrations change must be narrow compared with the “depletion layer” in which the carrier densities are nonuniform. (Typical plots of the carrier densities are superimposed on this figure in Figure 29.3.)

called the "doping profile." The term "junction" is used to refer both to the device as a whole and, more specifically, to the transition region about $x = 0$ in which the doping profile is nonuniform.

As we shall see below, the nonuniformity in impurity concentrations induces a nonuniformity in the densities $n_c(x)$ and $p_v(x)$ of conduction band electrons and valence band holes, which in turn gives rise to a potential $\phi(x)$. The region in which these carrier densities are nonuniform is known as the "depletion layer" (or "space-charge region"). The depletion layer can extend for a range of about 10^2 to 10^4 Å around the (generally more narrow) transition region in which the doping profile varies, as we shall see below. Within the depletion layer, except near its boundaries, the total density of carriers is very much less than it is in the homogeneous regions farther away from the transition region. The existence of a depletion layer is one of the crucial properties of the p - n junction. One of our main concerns will be to explain why such a layer is induced by the variation in impurity concentrations, and how its structure changes with the application of an external potential V .

For simplicity we shall consider here only "abrupt junctions," in which the transition region is so sharp that variation in impurity concentrations¹ can be represented by a single discontinuous change at $x = 0$:

$$\begin{aligned} N_d(x) &= \begin{cases} N_d & x > 0 \\ 0 & x < 0 \end{cases}, \\ N_a(x) &= \begin{cases} 0 & x > 0 \\ N_a & x < 0 \end{cases}. \end{aligned} \quad (29.1)$$

Abrupt junctions are not only conceptually the simplest, but also the type of greatest practical interest. How sharp the actual transition region must be made for (29.1) to give a reasonable model of a physical junction will emerge in the analysis below. We shall find that a junction may be regarded as abrupt if the transition region in the actual doping profile is small in extent compared with the depletion layer. In most cases this permits the transition region to extend for 100 Å or more. A junction that cannot be treated as abrupt is called a "graded junction."

THE SEMICLASSICAL MODEL

To calculate the response of an inhomogeneous semiconductor to an applied electrostatic potential, or even to compute the distribution of electric charge in the absence of an applied potential, one almost always uses the semiclassical model of Chapter 12. When a potential $\phi(x)$ is superimposed on the periodic potential of the crystal, the semiclassical model treats the electrons in the n th band as classical particles (i.e., as wave packets) governed by the Hamiltonian

$$H_n = \varepsilon_n \left(\frac{p}{\hbar} \right) - e\phi(x). \quad (29.2)$$

¹ It is not essential that there be only donor impurities in the n -type region and only acceptor impurities in the p -type region. It suffices for each impurity type to be the dominant one in its own region. In what follows N_d may be viewed as the excess density of donors over acceptors and N_a as the excess density of acceptors over donors.

Such a treatment is valid provided that the potential $\phi(x)$ varies sufficiently slowly. How slow this variation must be is, in general, a very difficult question to answer. At the very least, one requires that the change in electrostatic energy $e\Delta\phi$ over a distance of the order of the lattice constant be small compared with the band gap E_g , but the condition may well be even more stringent than this.² In the case of the p - n junction the potential ϕ has almost all of its spatial variation within the depletion layer. There, as we shall see, the energy $e\phi$ changes by about E_g over a distance that is typically a few hundred angstroms or more (so that the field in the depletion layer can be as large as 10^6 volts per meter). Although this satisfies the minimum necessary condition for the validity of the semiclassical model (the change in $e\phi$ over a lattice constant is no more than a fraction of a percent of E_g), the variation is strong enough that one cannot exclude the possibility that the semiclassical description may break down in the depletion layer. Thus one should bear in mind the possibility that the field in the depletion layer may be strong enough to induce tunneling of electrons from valence band to conduction band levels, leading to a conductivity considerably in excess of the semiclassical prediction.

Having issued this warning, however, we shall follow the general practice of assuming the validity of the semiclassical description, so that we may explore its consequences. Before describing the semiclassical theory of the currents that flow in a p - n junction in the presence of an applied potential, we first examine the case of the p - n junction in thermal equilibrium, in the absence of applied potentials and current flow.

THE p - n JUNCTION IN EQUILIBRIUM

We wish to determine the carrier densities and the electrostatic potential $\phi(x)$ induced by the nonuniform doping. We assume that nondegenerate conditions hold throughout the material, so that the carrier densities at each position x have the "Maxwellian" forms analogous to the densities (28.12) we found in the uniform case. In the non-uniform case, to derive the carrier densities at position x along the junction in the presence of a potential $\phi(x)$, the semiclassical procedure is simply to repeat the analysis for the uniform case, but using the semiclassical one-electron energy (29.2), in which each level is shifted by $-e\phi(x)$. Using the forms (28.3) of the $\mathcal{E}(\mathbf{k})$ appropriate to levels near the conduction band minimum or valence band maximum, we see that the effect of this is simply to shift the constants \mathcal{E}_c and \mathcal{E}_v by $-e\phi(x)$. Thus Eq. (28.12) for the equilibrium carrier densities is generalized to

$$\begin{aligned} n_c(x) &= N_c(T) \exp \left\{ - \frac{[\mathcal{E}_c - e\phi(x) - \mu]}{k_B T} \right\}, \\ p_v(x) &= P_v(T) \exp \left\{ - \frac{[\mu - \mathcal{E}_v + e\phi(x)]}{k_B T} \right\}. \end{aligned} \quad (29.3)$$

The potential $\phi(x)$ must be determined self-consistently, as that potential arising (via Poisson's equation) when the carrier densities have the forms (29.3). We examine

² A crude argument appropriate to metals is given in Appendix J. Analogous arguments (of comparable crudity) can be developed for semiconductors.

this problem in the special case (again, the case of major practical interest) in which far from the transition region on either side extrinsic conditions prevail, in which the impurities are fully "ionized" (see pages 583–584). Thus far away on the *n*-side the density of conduction band electrons is very nearly equal to the density N_d of donors, while far away on the *p*-side the density of valence band holes is very nearly equal to the density N_a of acceptors:

$$\begin{aligned} N_d &= n_c(\infty) = N_c(T) \exp \left\{ -\frac{[\varepsilon_c - e\phi(\infty) - \mu]}{k_B T} \right\}, \\ N_a &= p_v(-\infty) = P_v(T) \exp \left\{ -\frac{[\mu - \varepsilon_v + e\phi(-\infty)]}{k_B T} \right\}. \end{aligned} \quad (29.4)$$

Since the entire crystal is in thermal equilibrium, the chemical potential does not vary with position. In particular, the same value of μ appears in either of Eqs. (29.4); this immediately requires that the total potential drop across the junction be given by³

$$e\phi(\infty) - e\phi(-\infty) = \varepsilon_c - \varepsilon_v + k_B T \ln \left[\frac{N_d N_a}{N_c P_v} \right], \quad (29.5)$$

or

$$e \Delta\phi = E_g + k_B T \ln \left[\frac{N_d N_a}{N_c P_v} \right]. \quad (29.6)$$

An alternative way of representing the information in (29.3) and (29.6) is sometimes helpful. If we define a position-dependent "electrochemical potential" $\mu_e(x)$ by

$$\mu_e(x) = \mu + e\phi(x), \quad (29.7)$$

then we may write the carrier densities (29.3) as

$$\begin{aligned} n_c(x) &= N_c(T) \exp \left\{ -\frac{[\varepsilon_c - \mu_e(x)]}{k_B T} \right\}, \\ p_v(x) &= P_v(T) \exp \left\{ -\frac{[\mu_e(x) - \varepsilon_v]}{k_B T} \right\}. \end{aligned} \quad (29.8)$$

These have precisely the form of the relations (28.12) for a homogeneous semiconductor, except that the constant chemical potential μ is replaced by the electrochemical potential $\mu_e(x)$. Thus $\mu_e(\infty)$ is the chemical potential of a homogeneous *n*-type crystal whose properties are identical to those of the inhomogeneous crystal far on the *n*-side of the transition region, while $\mu_e(-\infty)$ is the chemical potential of a homogeneous *p*-type crystal identical to the inhomogeneous crystal far on the *p*-side. The relation (29.6) can equally well be written as⁴

$$e \Delta\phi = \mu_e(\infty) - \mu_e(-\infty). \quad (29.9)$$

³ The derivation of (29.5) requires the validity of (29.3) only far from the depletion layer, where ϕ is indeed slowly varying. It therefore holds even when the semiclassical model fails in the transition region.

⁴ This follows directly from (29.7). Equation (29.9) is sometimes summarized in the rule that the total potential drop is such as to bring the "Fermi levels at the two ends of the junction" into coincidence. This point of view is evidently inspired by the representation of Figure 29.2b.

Figure 29.2a shows the electrochemical potential plotted as a function of position along the p - n junction. We have assumed (as will be demonstrated below) that ϕ varies monotonically from one end to the other. Figure 29.2b shows an alternative representation of the same information in which the potential ϕ giving the position dependence in (29.3) is regarded as shifting ϵ_c (or ϵ_v) rather than μ . In either case, the significance of the diagrams is that at any particular position x along the junction, the carrier densities are those that would be found in a piece of homogeneous material with the impurity concentrations prevailing at x , and with a chemical potential that is positioned with respect to the band edges as shown in the vertical section of the diagrams at x .

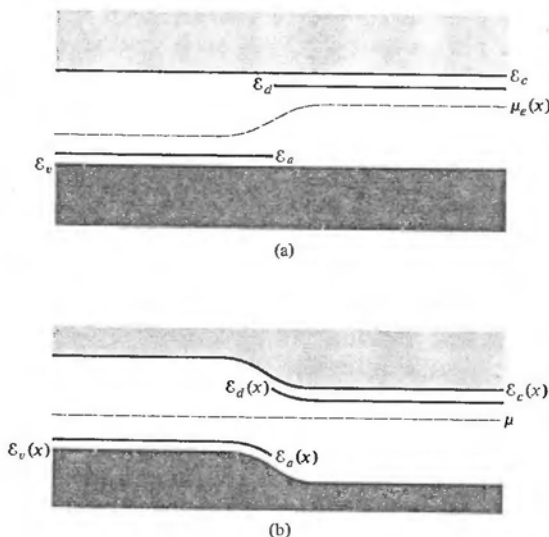


Figure 29.2

Two equivalent ways of representing the effect of the internal potential $\phi(x)$ on the electron and hole densities of a p - n junction. (a) The electrochemical potential $\mu_e(x) = \mu + e\phi(x)$ is plotted along the p - n junction. The carrier densities at any point x are those that would be found in a uniform semiconductor characterized by the fixed band and impurity energies ϵ_c , ϵ_v , ϵ_d , and ϵ_a , at a chemical potential equal to $\mu_e(x)$. (b) Here $\epsilon_c(x) = \epsilon_c - e\phi(x)$ is the energy of an electron wave packet localized about x formed from levels very near the conduction band minimum, and similarly for $\epsilon_v(x)$. The energies of the local impurity levels are $\epsilon_d(x) = \epsilon_d - e\phi(x)$ and $\epsilon_a(x) = \epsilon_a - e\phi(x)$. The (constant) chemical potential is also shown. The carrier densities at any point x are those that would be found in a uniform semiconductor characterized by band and impurity energies equal to $\epsilon_c(x)$, $\epsilon_d(x)$, $\epsilon_a(x)$, and $\epsilon_v(x)$ at the fixed chemical potential μ .

Equation (29.6) (or its equivalent form, (29.9)) serves as the boundary condition in a differential equation determining the potential $\phi(x)$. The differential equation is simply Poisson's equation,⁵

$$-\nabla^2\phi = -\frac{d^2\phi}{dx^2} = \frac{4\pi\rho(x)}{\epsilon}, \quad (29.10)$$

⁵ Here ϵ is the static dielectric constant of the semiconductor. The use of the macroscopic equation is possible because ϕ varies over the depletion layer, which is large on the interatomic scale.

relating the potential $\phi(x)$ to the charge distribution $\rho(x)$ giving rise to it. To express $\rho(x)$ in terms of ϕ and get a closed equation, we first note that if (as we have assumed) the impurities are fully ionized far from the junction, then they will remain fully ionized⁶ at all x . Consequently the charge density due to the impurities and the carriers is⁷

$$\rho(x) = e[N_d(x) - N_a(x) - n_c(x) + p_v(x)]. \quad (29.11)$$

When the carrier and impurity densities (29.3) and (29.1) are substituted into the form (29.11) for the charge density, and the result is substituted into Poisson's equation (29.10), one finds a nonlinear differential equation for $\phi(x)$ whose exact solution usually requires numerical techniques.⁸ However, a quite reasonable description of $\phi(x)$ may be had by exploiting the fact that the total change in $e\phi$ is of order $E_g \gg k_B T$. The relevance of this fact emerges when we combine (29.3) and (29.4) to write

$$\begin{aligned} n_c(x) &= N_d e^{-e[\phi(x) - \phi(x)]/k_B T}, \\ p_v(x) &= N_a e^{-e[\phi(x) - \phi(x)]/k_B T}. \end{aligned} \quad (29.12)$$

Suppose that the change in ϕ occurs within a region $-d_p \leq x \leq d_n$. Outside of this region, ϕ has its asymptotic value, and therefore $n_c = N_d$ on the n side, $p_v = N_a$ on the p side, and $\rho = 0$. Within the region, except quite near the boundaries, $e\phi$ differs by many $k_B T$ from its asymptotic value, so $n_c \ll N_d$, $p_v \ll N_a$. Thus, except in the vicinity of $x = -d_p$ and $x = d_n$, the charge density (29.11) between $-d_p$ and d_n is quite accurately given by $\rho(x) = e[N_d(x) - N_a(x)]$, there being no appreciable carrier charge to cancel the charges of the "ionized" impurities. The points $x = -d_p$ and $x = d_n$ therefore mark the boundaries of the depletion layer.

Combining these observations, and using the form (29.1) for the impurity densities, we find that except for x just greater than $-d_p$ or just less than d_n , Poisson's equation is well approximated by

$$\phi''(x) = \left. \begin{array}{ll} 0, & x > d_n, \\ \frac{-4\pi e N_d}{\epsilon}, & d_n > x > 0, \\ \frac{4\pi e N_a}{\epsilon}, & 0 > x > -d_p, \\ 0, & -d_p > x. \end{array} \right\} \quad (29.13)$$

⁶ If ϕ is monotonic (as we shall find below) this follows from the fact that the degree of ionization of an impurity increases, the farther the chemical potential is from the impurity level. See Figure 29.2 and Eqs. (28.32) and (28.34).

⁷ The density of holes on the far n -side has the very small value $p_v(\infty) = n_i^2/N_d$ required by the law of mass action. However, the density of electrons on the far n -side actually exceeds N_d by this same small amount so as to insure that $n_c(\infty) - p_v(\infty) = N_d$. In computing the total charge density, if we ignore this small correction in n_c (as we have done in writing (29.4)), then we should also ignore the small compensating density of holes on the far n -side. Similar remarks apply to the small concentration of electrons on the far p -side. These "minority carrier densities" have negligible effect on the total balance of charge. We shall see below, however, that they play an important role in determining the flow of currents in the presence of an applied potential.

⁸ Some aspects of that equation are investigated in Problem 1.

This immediately integrates to give

$$\phi(x) = \left\{ \begin{array}{ll} \phi(\infty), & x > d_n, \\ \phi(\infty) - \left(\frac{2\pi e N_d}{\epsilon}\right)(x - d_n)^2, & d_n > x > 0, \\ \phi(-\infty) + \left(\frac{2\pi e N_a}{\epsilon}\right)(x + d_p)^2, & 0 > x > -d_p, \\ \phi(-\infty), & x < -d_p. \end{array} \right\} \quad (29.14)$$

The boundary conditions (continuity of ϕ and its first derivative) are explicitly obeyed by the solution (29.14) at $x = -d_p$ and $x = d_n$. Requiring them to hold at $x = 0$ gives two additional equations that determine the lengths d_n and d_p . Continuity of ϕ' at $x = 0$ implies that

$$N_d d_n = N_a d_p \quad (29.15)$$

which is just the condition that the excess of positive charge on the n -side of the junction be equal to the excess of negative charge on the p -side. Continuity of ϕ at $x = 0$ requires that

$$\left(\frac{2\pi e}{\epsilon}\right)(N_d d_n^2 + N_a d_p^2) = \phi(\infty) - \phi(-\infty) = \Delta\phi. \quad (29.16)$$

Together with (29.15) this determines the lengths d_n and d_p :

$$d_{n,p} = \left\{ \frac{(N_a/N_d)^{\pm 1} \epsilon \Delta\phi}{(N_d + N_a) 2\pi e} \right\}^{1/2} \quad (29.17)$$

To estimate the sizes of these lengths we may write Eq. (29.17) in the numerically more convenient form

$$d_{n,p} = 105 \left\{ \frac{(N_a/N_d)^{\pm 1}}{10^{-18}(N_d + N_a)} [\epsilon e \Delta\phi]_{\text{eV}} \right\}^{1/2} \text{ \AA}. \quad (29.18)$$

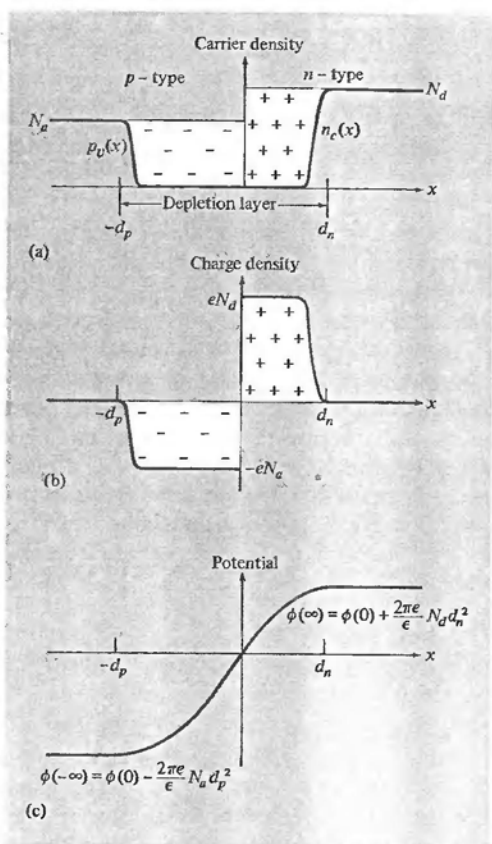
The quantity $\epsilon e \Delta\phi$ is typically of order 1 eV, and since typical impurity concentrations are in the range from 10^{14} to 10^{18} per cubic centimeter, the lengths d_n and d_p , which give the extent of the depletion layer, will generally be from 10^4 to 10^2 Å. The field within the depletion layer is of order $\Delta\phi/(d_n + d_p)$, and for d 's of this size is therefore in the range from 10^5 to 10^7 volts per meter, for an energy gap of 0.1 eV.

The resulting picture of the depletion layer is shown in Figure 29.3. The potential ϕ varies monotonically through the layer, as asserted above. Except at the boundaries of the layer, the carrier concentrations are negligible compared with the impurity concentrations, so the charge density is that of the ionized impurities. Outside of the depletion layer the carrier concentrations balance the impurity concentrations, and the charge density is zero.

The mechanism establishing such a region of sharply reduced carrier densities is relatively simple. Suppose that one initially were able to impose carrier concentrations that gave charge neutrality at every point in the crystal. Such a configuration could not be maintained, for electrons would begin to diffuse from the n -side (where their concentration was high) to the p -side (where their concentration was very low), and

Figure 29.3

(a) Carrier densities, (b) charge density, and (c) potential $\phi(x)$ plotted vs. position across an abrupt p - n junction. In the analysis in the text the approximation was made that the carrier densities and charge density are constants except for discontinuous changes at $x = -d_p$ and $x = d_n$. More precisely (see Problem 1), these quantities undergo rapid change over regions just within the depletion layer whose extent is a fraction of order $(k_B T/E_g)^{1/2}$ of the total extent of the depletion layer. The extent of the depletion layer is typically from 10^2 to 10^4 Å.



holes would diffuse in the opposite direction. As this diffusion continued, the resulting transfer of charge would build up an electric field opposing further diffusive currents, until an equilibrium configuration was reached in which the effect of the field on the currents precisely canceled the effect of diffusion. Because the carriers are highly mobile, in this equilibrium configuration the carrier densities are very low wherever the field has an appreciable value. This is precisely the state of affairs depicted in Figure 29.3.

ELEMENTARY PICTURE OF RECTIFICATION BY A p - n JUNCTION

We now consider the behavior of a p - n junction when an external voltage V is applied. We shall take V to be positive if its application raises the potential of the p -side with respect to the n -side. When $V = 0$ we found above that there is a depletion layer some 10^2 to 10^4 Å in extent about the transition point where the doping changes from p -type to n -type, in which the density of carriers is reduced greatly below its value in the homogeneous regions farther away. Because of its greatly reduced carrier

density, the depletion layer will have a much higher electrical resistance than the homogeneous regions, and the whole device can therefore be viewed as a series circuit in which a relatively high resistance is sandwiched between two relatively low resistances. When a potential V is applied across such a circuit, almost all of the potential drop will occur across the region of high resistance. Thus even in the presence of an applied potential V , we expect that the potential $\phi(x)$ along the device will vary appreciably only within the depletion layer. When $V = 0$ we found that $\phi(x)$ rose from the p -side of the depletion layer to the n -side by the amount (which we now denote by $(\Delta\phi)_0$) given by Eq. (29.6), so we conclude that when $V \neq 0$ the change in potential across the depletion layer is modified to

$$\Delta\phi = (\Delta\phi)_0 - V. \quad (29.19)$$

Associated with this change in potential drop across the depletion layer, there is a change in the size of the layer. The lengths d_n and d_p giving the extent of the layer on the n - and p -sides of the junction are determined by Eqs. (29.15) and (29.16), which use only the value of the total potential drop across the layer, and the assumption that the carrier densities are greatly reduced throughout almost all of the layer. We shall find below that this assumption remains valid when $V \neq 0$, and therefore d_n and d_p continue to be given by Eq. (29.17) provided that we take the value of $\Delta\phi$ to be $(\Delta\phi)_0 - V$. Since d_n and d_p vary as $(\Delta\phi)^{1/2}$ according to Eq. (29.17), we conclude that when $V \neq 0$,

$$d_{n,p}(V) = d_{n,p}(0) \left[1 - \frac{V}{(\Delta\phi)_0} \right]^{1/2}. \quad (29.20)$$

This behavior of ϕ and the extent of the depletion layer are illustrated in Figure 29.4.

To deduce the dependence on V of the current that flows when a p - n junction is "biased" by the application of an external voltage, we must consider separately the currents of electrons and holes. Throughout the discussion that follows we shall use the symbol J for number current densities and j for electrical current densities, so that

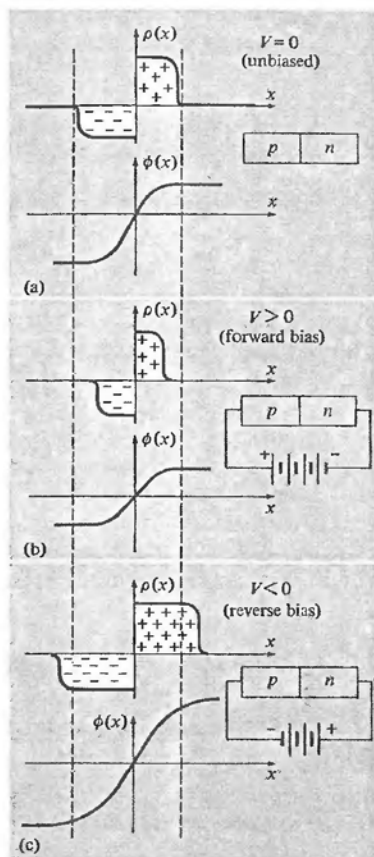
$$j_e = -eJ_e, \quad j_h = eJ_h. \quad (29.21)$$

When $V = 0$, both J_e and J_h vanish. This does not, of course, mean that no individual carriers flow across the junction, but only that as many electrons (or holes) flow in one direction as in the other. When $V \neq 0$, this balance is disrupted. Consider, for example, the current of holes across the depletion layer. This has two components:

1. A hole current flows from the n - to the p -side of the junction, known as the hole *generation current*. As the name indicates, this current arises from holes that are generated just on the n -side of the depletion layer by the thermal excitation of electrons out of valence band levels. Although the density of such holes on the n -side ("minority carriers") is minute compared with the density of electrons ("majority carriers"), they play an important role in carrying current across the junction. This is because any such hole that wanders into the depletion layer is immediately swept over to the p -side of the junction by the strong electric field

Figure 29.4

The charge density ρ and potential ϕ in the depletion layer (a) for the unbiased junction, (b) for the junction with $V > 0$ (forward bias), and (c) for the junction with $V < 0$ (reverse bias). The positions $x = d_n$ and $x = -d_p$ that mark the boundaries of the depletion layer when $V = 0$ are given by the dashed lines. The depletion layer and change in ϕ are reduced by a forward bias and increased by a reverse bias.



that prevails within the layer. The resulting generation current is insensitive to the size of the potential drop across the depletion layer, since any hole, having entered the layer from the n -side, will be swept through to the p -side.⁹

2. A hole current flows from the p - to the n -side of the junction, known as the hole *recombination current*.¹⁰ The electric field in the depletion layer acts to oppose such a current, and only holes that arrive at the edge of the depletion layer with a thermal energy sufficient to surmount the potential barrier will contribute to

⁹ The density of holes giving rise to the hole generation current will also be insensitive to the size of V , provided that eV is small compared with E_g , for this density is entirely determined by the law of mass action and the density of electrons. The latter density differs only slightly from the value N_c outside of the depletion layer when eV is small compared with E_g , as will emerge from the more detailed analysis below.

¹⁰ So named because of the fate suffered by such holes upon arriving on the n -side of the junction, where one of the abundant electrons will eventually drop into the empty level that constitutes the hole.

the recombination current. The number of such holes is proportional to $e^{-e\Delta\phi/k_B T}$, and therefore¹¹

$$J_h^{\text{rec}} \propto e^{-e[(\Delta\phi)_0 - V]/k_B T}. \quad (29.22)$$

In contrast to the generation current, the recombination current is highly sensitive to the applied voltage V . We can compare their magnitudes by noting that when $V = 0$ there can be no net hole current across the junction:

$$J_h^{\text{rec}}|_{V=0} = J_h^{\text{gen}}. \quad (29.23)$$

Taken together with Eq. (29.22), this requires that

$$J_h^{\text{rec}} = J_h^{\text{gen}} e^{eV/k_B T}. \quad (29.24)$$

The total current of holes flowing from the p - to the n -side of the junction is given by the recombination current minus the generation current:

$$J_h = J_h^{\text{rec}} - J_h^{\text{gen}} = J_h^{\text{gen}}(e^{eV/k_B T} - 1). \quad (29.25)$$

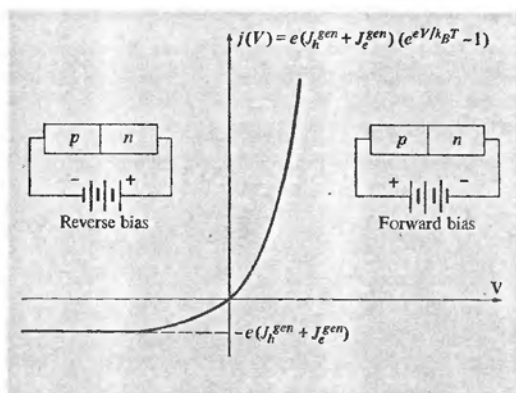
The same analysis applies to the components of the electron current, except that the generation and recombination currents of electrons flow oppositely to the corresponding currents of holes. Since, however, the electrons are oppositely charged, the electrical generation and recombination currents of electrons are parallel to the electrical generation and recombination currents of holes. The total electrical current density is thus:

$$j = e(J_h^{\text{gen}} + J_e^{\text{gen}})(e^{eV/k_B T} - 1). \quad (29.26)$$

This has the highly asymmetric form characteristic of rectifiers, as shown in Figure 29.5.

Figure 29.5

Current vs. applied voltage V for a p - n junction. The relation is valid for eV small compared with the energy gap, E_g . The saturation current ($eJ_h^{\text{gen}} + eJ_e^{\text{gen}}$) varies with temperature as $e^{-E_g/k_B T}$, as established below.



¹¹ In assuming that (29.22) gives the dominant dependence of the hole recombination current on V , we are assuming that the density of holes just on the p -side of the depletion layer differs only slightly from N_a . We shall find that this is also the case provided that eV is small compared with the energy gap E_g .

GENERAL PHYSICAL ASPECTS OF THE NONEQUILIBRIUM CASE

The foregoing discussion provides no estimate of the size of the prefactor $e(J_h^{\text{gen}} + J_e^{\text{gen}})$ appearing in (29.26). In addition, in the nonequilibrium ($V \neq 0$) case the local carrier densities will not in general be determined by the local potential ϕ through the simple equilibrium Maxwellian relations (29.3). In the nonequilibrium case it requires further analysis to construct a picture of the carrier densities in the neighborhood of the transition region that is comparable in detail to the picture we gave for the equilibrium case.

In this more detailed approach it is not especially helpful to resolve the electron and hole currents across the junction into generation and recombination currents. Instead, at each point x (both inside and outside the depletion layer) we shall write equations relating the total electron and hole currents, $J_e(x)$ and $J_h(x)$, the electron and hole densities, $n_c(x)$ and $p_v(x)$, and the potential $\phi(x)$ (or, equivalently, the electric field, $E(x) = -d\phi(x)/dx$). We shall find five such equations, which will enable us, in principle, to find these five quantities. This method is a direct generalization of the approach we followed in our analysis of the equilibrium ($V = 0$) case. In equilibrium the electron and hole currents vanish, there are only three unknowns, and the three equations we used were Poisson's equation, and the two equations (29.3) that relate $n_c(x)$ and $p_v(x)$ to $\phi(x)$ in thermal equilibrium. Thus the nonequilibrium problem can be viewed as that of finding the appropriate equations to replace the equilibrium relation (29.3), when $V \neq 0$ and currents flow.

We first observe that in the presence of both an electric field and a carrier density gradient, the carrier current density can be written as the sum of a term proportional to the field (the *drift current*) and a term proportional to the density gradient (the *diffusion current*):

$$\begin{aligned} J_e &= -\mu_n n_c E - D_n \frac{dn_c}{dx}, \\ J_h &= \mu_p p_v E - D_p \frac{dp_v}{dx}. \end{aligned} \quad (29.27)$$

The positive¹² proportionality constants μ_n and μ_p appearing in Eq. (29.27) are known as the electron and hole *mobilities*. We have introduced the mobilities, rather than writing the drift current in terms of conductivities, to make explicit the manner in which the drift current depends on the carrier densities. If only electrons at uniform density are present, then $\sigma E = j = -eJ_e = e\mu_n E$. Using the Drude form $\sigma = ne^2\tau/m$ for the conductivity (Eq. (1.6)) we find that

$$\mu_n = \frac{e\tau_n^{\text{coll}}}{m_n}, \quad (29.28)$$

¹² The signs in Eq. (29.27) have been chosen to make the mobilities positive; the hole drift current is along the field, and the electron drift current is opposite to the field.

and, similarly,

$$\mu_p = \frac{e\tau_p^{\text{coll}}}{m_p}, \quad (29.29)$$

where m_n and m_p are the appropriate effective masses, and τ_n^{coll} and τ_p^{coll} are the carrier collision times.¹³

The positive¹⁴ proportionality constants D_n and D_p appearing in Eq. (29.27) are known as the electron and hole *diffusion constants*. They are related to the mobilities by the *Einstein relations*:¹⁵

$$\boxed{\mu_n = \frac{eD_n}{k_B T}, \quad \mu_p = \frac{eD_p}{k_B T}.} \quad (29.30)$$

The Einstein relations follow directly from the fact that the electron and hole currents must vanish in thermal equilibrium: Only if the mobilities and diffusion constants are related by (29.30) will the currents given by (29.27) be zero when the carrier densities have the equilibrium form (29.3)¹⁶ (as is easily verified by direct substitution of (29.3) into (29.27)).

The relation (29.27) giving the currents in terms of the density gradients and field, together with the forms (29.28)–(29.30) for the mobilities and diffusion constants, can also be derived directly from the kind of simple kinetic argument used in Chapter 1 (see Problem 2).

Note that in thermal equilibrium, Eq. (29.27) and the conditions $J_e = J_h = 0$ contain all information necessary to determine the carrier densities, for when the currents vanish we may integrate Eq. (29.27) to rederive (with the aid of the Einstein relations (29.30)) the thermal equilibrium densities (29.3). When $V \neq 0$ and currents flow, we require a further equation, which can be viewed as the generalization to the nonequilibrium case of the equilibrium conditions of vanishing currents. If the numbers of carriers were conserved, the required generalization would simply be the equations of continuity,

$$\begin{aligned} \frac{\partial n_c}{\partial t} &= -\frac{\partial J_e}{\partial x}, \\ \frac{\partial p_v}{\partial t} &= -\frac{\partial J_h}{\partial x}, \end{aligned} \quad (29.31)$$

which express the fact that the change in the number of carriers in a region is entirely determined by the rate at which carriers flow into and out of the region. However, carrier numbers are not conserved. A conduction band electron and a valence band

¹³ In semiconductors there is another lifetime of fundamental importance (see below), the *recombination time*. The superscript “coll” has been affixed to the collision mean free times to distinguish them from the recombination times.

¹⁴ They are positive because the diffusion current flows from high- to low-density regions. In zero field, Eq. (29.27) is sometimes known as Fick’s law.

¹⁵ The Einstein relations are very general, arising in any treatment of charged particles that obey Maxwell-Boltzmann statistics, such as the ions in an electrolytic solution.

¹⁶ The generalization of (29.30) to the degenerate case is described in Problem 3.

hole can be *generated* by the thermal excitation of an electron out of a valence band level. Furthermore a conduction band electron and a valence band hole can *recombine* (i.e., the electron can drop into the empty level that is the hole), resulting in the disappearance of one carrier of each type. Terms must be added to the continuity equations describing these other ways in which the number of carriers in a region can change:

$$\begin{aligned}\frac{\partial n_c}{\partial t} &= \left(\frac{dn_c}{dt}\right)_{g-r} - \frac{\partial J_e}{\partial x}, \\ \frac{\partial p_v}{\partial t} &= \left(\frac{dp_v}{dt}\right)_{g-r} - \frac{\partial J_h}{\partial x}.\end{aligned}\quad (29.32)$$

To determine the forms of $(dn_c/dt)_{g-r}$ and $(dp_v/dt)_{g-r}$ we note that generation and recombination act to restore thermal equilibrium when the carrier densities deviate from their equilibrium values. In regions where n_c and p_v exceed their equilibrium values, recombination occurs faster than generation, leading to a decrease in the carrier densities, while in regions where they fall short of their equilibrium values, generation occurs faster than recombination, leading to an increase in the carrier densities. In the simplest models these processes are described by electron and hole lifetimes,¹⁷ τ_n and τ_p . The rate at which each carrier density changes due to recombination and generation is set proportional to its deviation from the form determined by the other carrier density and the law of mass action (28.24):

$$\begin{aligned}\left(\frac{dn_c}{dt}\right)_{g-r} &= -\frac{(n_c - n_c^0)}{\tau_n}, \\ \left(\frac{dp_v}{dt}\right)_{g-r} &= -\frac{(p_v - p_v^0)}{\tau_p},\end{aligned}\quad (29.33)$$

where $n_c^0 = n_i^2/p_v$, and $p_v^0 = n_i^2/n_c$.

To interpret these equations, note that the first, for example, expresses the change in electron carrier density due to generation and recombination in an infinitesimal time dt as

$$n_c(t + dt) = \left(1 - \frac{dt}{\tau_n}\right)n_c(t) + \left(\frac{dt}{\tau_n}\right)n_c^0. \quad (29.34)$$

The first term on the right of Eq. (29.34) expresses the destruction, through recombination, of a fraction dt/τ_n of the electron carriers; i.e., τ_n is the average electronic lifetime before recombination occurs. The second term on the right expresses the creation through thermal generation of n_c^0/τ_n electron carriers per unit volume, per unit time. Note that, as required, Eqs. (29.33) give carrier densities that decrease when they exceed their equilibrium values, increase when they are less than their equilibrium values, and do not change when they are equal to their equilibrium values.

The lifetimes τ_n and τ_p are generally much longer than overall electron or hole collision times, τ_n^{coll} and τ_p^{coll} , for the recombination (or generation) of an electron

¹⁷ Also known as "recombination times." Conservation of total electric charge requires that the recombination rates be proportional to the densities of the other carrier type: $(1/\tau_n)/(1/\tau_p) = p_v/n_c$.

and a hole is an interband transition (electron goes from valence to conduction band (generation) or conduction band to valence band (recombination)). Ordinary collisions, which conserve the number of carriers, are intraband transitions. Reflecting this, typical lifetimes range between 10^{-3} and 10^{-8} second, while the collision times are similar to those found in metals—i.e., 10^{-12} or 10^{-13} second.

In the presence of a static external potential the p - n junction, though not in thermal equilibrium, is in a steady state; i.e., the carrier densities will be constant in time: $dn_c/dt = dp_v/dt = 0$. Using this fact and the forms (29.33) for the rates at which recombination and generation change the carrier densities, we find that the continuity equation (29.32) requires

$$\boxed{\begin{aligned} \frac{dJ_e}{dx} + \frac{n_c - n_c^0}{\tau_n} &= 0, \\ \frac{dJ_h}{dx} + \frac{p_v - p_v^0}{\tau_p} &= 0. \end{aligned}} \quad (29.35)$$

These are the equations that replace the equilibrium conditions $J_e = J_h = 0$, when $V \neq 0$.

One very important application of Eqs. (29.35) and (29.27) is in regions where the electric field E is negligibly small and the majority carrier density is constant. In that case the minority carrier drift current can be ignored compared with the minority carrier diffusion current, and Eqs. (29.27) and (29.35) reduce to a single equation for the minority carrier density with a constant recombination time:

$$\boxed{\begin{aligned} D_n \frac{d^2 n_c}{dx^2} &= \frac{n_c - n_c^0}{\tau_n}, \\ D_p \frac{d^2 p_v}{dx^2} &= \frac{p_v - p_v^0}{\tau_p}. \end{aligned}} \quad (E \approx 0) \quad (29.36)$$

The solutions to these equations vary exponentially in x/L , where the lengths

$$L_n = (D_n \tau_n)^{1/2}, \quad L_p = (D_p \tau_p)^{1/2}, \quad (29.37)$$

are known as the electron and hole *diffusion lengths*. Suppose, for example, (to take a case that will be of some importance below) that we are in the region of uniform potential on the n -side of the depletion layer, so that the equilibrium density p_v^0 has the constant value $p_v(\infty) = n_i^2/N_d$. If the density of holes is constrained to have the value $p_v(x_0) \neq p_v(\infty)$ at a point x_0 , then the solution to Eq. (29.36) for $x \geq x_0$ is

$$p_v(x) = p_v(\infty) + [p_v(x_0) - p_v(\infty)]e^{-(x-x_0)/L_p}. \quad (29.38)$$

Thus the diffusion length is a measure of the distance it takes for the density to relax back to its equilibrium value.

One would expect that the distance L over which a deviation from equilibrium density can be maintained would be roughly the distance a carrier can travel before undergoing recombination. This is not immediately obvious from the forms (29.37) for the diffusion lengths L_n and L_p , but it is revealed when one rewrites (29.37) using (a) the Einstein relations (29.30) between the diffusion constant and the mobility,

(b) the Drude form (29.28), or (29.29), for the mobility, (c) the relation $\frac{1}{2}mv_{th}^2 = \frac{3}{2}k_B T$ between the mean square carrier velocity and the temperature under nondegenerate conditions, and (d) the definition $\ell = v_{th}\tau^{coll}$ of the carrier mean free path between collisions. Making these substitutions one finds:

$$\begin{aligned} L_n &= \left(\frac{\tau_n}{3\tau_n^{coll}} \right)^{1/2} \ell_n, \\ L_p &= \left(\frac{\tau_p}{3\tau_p^{coll}} \right)^{1/2} \ell_p. \end{aligned} \quad (29.39)$$

Assuming that the direction of a carrier is random after each collision, a series of N collisions can be viewed as a random walk of step length ℓ . It is easily shown¹⁸ that in such a walk the total displacement is $N^{1/2}\ell$. Since the number of collisions a carrier can undergo in a recombination time is the ratio of the recombination time to the collision time, Eq. (29.39) does indeed show that the diffusion length measures the distance a carrier can go before undergoing recombination.

Using the typical values given on page 604 for the collision time and the (very much longer) recombination time, we find that (29.39) gives a diffusion length that can be between 10^2 and 10^5 mean free paths.

We can estimate the size of the generation currents that appear in the I - V relation (29.26), in terms of the diffusion lengths and carrier lifetimes. We first note that, by definition of the lifetime, holes are created by thermal generation at a rate p_v^0/τ_p per unit volume. Such a hole stands an appreciable chance of entering the depletion layer (and then being swiftly swept across to the n -side) before undergoing recombination, provided that it is created within a diffusion length L_p of the boundary of the depletion layer. Therefore the flow of thermally generated holes per unit area into the depletion layer per second will be of order $L_p p_v^0/\tau_p$. Since $p_v^0 = n_i^2/N_d$, we have

$$J_h^{gen} = \left(\frac{n_i^2}{N_d} \right) \frac{L_p}{\tau_p}, \quad (29.40)$$

and, similarly,

$$J_e^{gen} = \left(\frac{n_i^2}{N_a} \right) \frac{L_n}{\tau_n}. \quad (29.41)$$

The sum of the currents appearing in (29.40) and (29.41) is known as the *saturation current*, since it is the maximum current that can flow through the junction when V is negative ("reversed bias"). Because the temperature dependence of n_i^2 is dominated by the factor $e^{-E_g/k_B T}$ (Eq. (28.19)), the saturation current is strongly temperature-dependent.

A MORE DETAILED THEORY OF THE NONEQUILIBRIUM p - n JUNCTION

Using the concepts of drift and diffusion currents we can give a more detailed description of the behavior of the p - n junction when $V \neq 0$. The equilibrium p - n

¹⁸ See for example, F. Reif, *Fundamentals of Statistical and Thermal Physics*, McGraw-Hill, New York, 1965, p. 16.

junction has two characteristic regions: the depletion layer, in which the electric field, space charge, and carrier density gradients are large, and the homogeneous regions outside of the depletion layer, in which they are quite small. In the non-equilibrium case the position beyond which the electric field and space charge are small differs from the position beyond which the carrier density gradients are small. Thus when $V \neq 0$, the p - n junction is characterized not by two, but by three different regions (described compactly in Table 29.1):

1. **The Depletion Layer** As in the equilibrium case, this is a region in which the electric field, space charge, and carrier density gradients are all large. When $V \neq 0$, according to Eq. (29.20) the depletion layer is narrower than or wider than is the case for $V = 0$ depending on whether V is positive (forward bias) or negative (reverse bias).
2. **The Diffusion Regions** These are regions (extending a distance of the order of a diffusion length out from the boundaries of the depletion layer) in which the electric field and space charge are small, but the carrier density gradients remain appreciable (though not as large as in the depletion layer).
3. **The Homogeneous Regions** Beyond the diffusion regions the electric field, space charge, and carrier density gradients are all very small, as in the equilibrium homogeneous regions.

The diffusion region (2) is not present in the equilibrium case. It arises when $V \neq 0$ for the following reason:

In equilibrium ($V = 0$) the change in carrier densities across the depletion layer is just enough to join the homogeneous equilibrium values on the high-density sides ($n_e(\infty) = N_d$, $p_h(-\infty) = N_a$), to the homogeneous equilibrium values on the low-density sides¹⁹ ($n_e(-\infty) = n_i^2/N_a$, $p_h(+\infty) = n_i^2/N_d$). When $V \neq 0$, however, we have noted that the extent of the depletion layer and the size of the potential drop across the layer differ from their equilibrium values. Consequently (as we shall see explicitly below) the change in carrier densities across the layer can no longer fit the difference in the homogeneous equilibrium values appropriate to the two sides, and a further region must appear in which the carrier densities relax from their values at the boundaries of the depletion layer to the values appropriate to the more remote homogeneous region (Figure 29.6).

Table 29.1 summarizes these properties, and also indicates the characteristic behavior of the electron and hole drift and diffusion currents in each of the three regions, when a current j flows in the junction²⁰:

1. *In the depletion layer* there are both drift and diffusion currents. In the equilibrium case they are equal and opposite for each carrier type, yielding no net currents of

¹⁹ In discussing the equilibrium case above, we approximated the minority carrier densities (i.e., the homogeneous equilibrium values on the low-density side) by zero (cf. note 7). This was appropriate because we were describing only the space-charge density, to which the minority carriers make a minute contribution compared with the majority carriers. The contribution of the minority carriers to the current, however, is not negligible, and it is necessary for us to use the values quoted here (determined by the values of the majority carrier densities and the law of mass action).

²⁰ In the steady state the total electric current is uniform along the junction: j cannot depend on x .

Table 29.1
THE THREE CHARACTERISTIC REGIONS IN A BIASED p - n JUNCTION ^a

	HOMOGENEOUS p -TYPE	DIFFUSION REGION $O(L_p)$	DEPLETION LAYER $d_p \rightarrow d_n$	DIFFUSION REGION $O(L_n)$	HOMOGENEOUS n -TYPE
ELECTRIC FIELD OR SPACE CHARGE					
$\nabla p, \nabla n$	SMALL	SMALL	LARGE	SMALL	SMALL
p	SMALL	LARGE	LARGE	SMALL	SMALL
n		SMALL		LARGE	
j_h^{diff}	$\approx j$	$O(j)$	$\gg j$	≈ 0	≈ 0
$j_h^{\text{diffusion}}$	≈ 0	$O(j)$	$\gg j$	$O(j)$	≈ 0
$j_e^{\text{diffusion}}$	≈ 0	$O(j)$	$\gg j$	$O(j)$	≈ 0
j_e^{diff}	≈ 0	≈ 0	$\gg j$	$O(j)$	$\approx j$

^a The positions and extent of the regions are indicated at the top of the table. The column beneath each region gives the orders of magnitude of the important physical quantities.

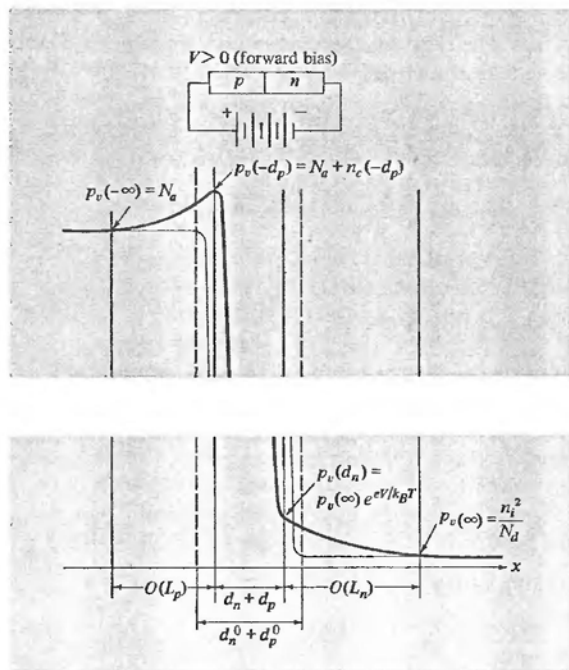


Figure 29.6

The density of holes (heavy curve) along a p - n junction with $V > 0$ (forward bias). The vertical solid lines give the boundaries of the depletion layer and diffusion regions. Note the break in vertical scale. For comparison, the density of holes when $V = 0$ (unbiased junction) is shown as a light curve, together with the boundaries of the depletion layer in the unbiased case (dashed vertical lines). The density of electrons behaves in a similar manner. When V is negative (reverse bias), the density of holes falls below its asymptotic value in the diffusion region. Note that although the excess density in the biased case over the unbiased case has the same size in both diffusion regions, on the p -side it represents a minute percentage change in the carrier density, while on the n -side it is a very large percentage change.

electrons or holes. In the nonequilibrium case the net current flowing across the depletion layer results from a slight imbalance between the drift and diffusion currents of each carrier type; i.e., the drift and diffusion currents are separately quite large compared with the total current. Once we have constructed a complete picture of the currents flowing in the junction it is easy to verify this explicitly (Problem 4). It is a consequence of the very large electric field and density gradients in the diffusion layer (which more than compensate for the very low carrier densities).

2. *In the diffusion regions*, the carrier densities are nearer to the values they have in the homogeneous regions. The majority carrier density has become so large that its drift current is appreciable, even though the field is now very small; the minority carrier drift current is quite negligible in comparison. Because the carrier densities continue to vary in the diffusion regions, both diffusion currents (proportional not to the density, but its gradient) are appreciable. Typically, all currents in the diffusion region except the negligible minority carrier drift current are of order j .
3. *In the homogeneous regions* the diffusion currents are negligible, and the entire current is carried by the majority carrier drift current.

Given this picture of the individual carrier drift and diffusion currents, we can readily calculate the total current j flowing in the junction for a given value of V . To simplify the analysis we make one further assumption.²¹ We assume that the passage of carriers across the depletion layer is so swift that negligible generation and recombination occur within the layer. If this is so, then the total currents of electrons and holes, J_e and J_h , will be constant across the depletion layer in the steady state. Consequently, in the expression $j = -eJ_e + eJ_h$ for the total current, we may separately evaluate J_e and J_h at whatever points along the depletion layer it is most convenient to do so. The most convenient point for the electron current is at the boundary between the depletion layer and the diffusion region on the p -side, and the most convenient point for the hole current is at the other boundary.²² We therefore write

$$j = -eJ_e(-d_p) + eJ_h(d_n). \quad (29.42)$$

This representation is useful because at the boundaries between the depletion layer and the diffusion regions the minority carrier currents are purely diffusive (see Table 29.1). Consequently, if we could calculate the position dependence of the minority carrier densities within the diffusion regions, we could immediately compute their currents, using Eq. (29.27) (with $E = 0$):

$$\begin{aligned} J_e(-d_p) &= -D_n \left. \frac{dn_c}{dx} \right|_{x=-d_p}, \\ J_h(d_n) &= -D_p \left. \frac{dp_v}{dx} \right|_{x=d_n}. \end{aligned} \quad (29.43)$$

Because, however, the minority carrier drift currents are negligible in the diffusion regions, the minority carrier densities satisfy the diffusion equation (29.36). If we let $p_v(d_n)$ be the density of holes at the boundary of the depletion layer on the n -side, and if we note that far from that boundary on the n -side p_v approaches the value $p_v(\infty) = n_i^2/N_d$, then the solution (29.38) to the diffusion equation (29.36) is

$$p_v(x) = \frac{n_i^2}{N_d} + \left[p_v(d_n) - \frac{n_i^2}{N_d} \right] e^{-(x-d_n)/L_p}, \quad x \geq d_n. \quad (29.44)$$

Similarly, the electron density within the diffusion region on the p -side is given by

$$n_c(x) = \frac{n_i^2}{N_a} + \left[n_c(-d_p) - \frac{n_i^2}{N_a} \right] e^{(x+d_p)/L_n}, \quad x \leq -d_p. \quad (29.45)$$

Substituting these densities into (29.43) we find that the minority carrier currents at the boundaries of the depletion layer are

$$\begin{aligned} J_e(-d_p) &= -\frac{D_n}{L_n} \left[n_c(-d_p) - \frac{n_i^2}{N_a} \right], \\ J_h(d_n) &= \frac{D_p}{L_p} \left[p_v(d_n) - \frac{n_i^2}{N_d} \right], \end{aligned} \quad (29.46)$$

²¹ Also generally the case. When the assumption fails, the full set of equations must be integrated across the depletion layer.

²² The more elementary picture of rectification given above also focused on the electron current originating on the hole side of the junction, and vice versa.

so that the total current, (29.42), is

$$j = \frac{eD_n}{L_n} \left[n_c(-d_p) - \frac{n_i^2}{N_a} \right] + \frac{eD_p}{L_p} \left[p_v(d_n) - \frac{n_i^2}{N_d} \right]. \quad (29.47)$$

It only remains for us to find the amounts by which the minority carrier densities differ from their homogeneous equilibrium values at the boundaries of the depletion layer. In equilibrium we found the variation in carrier densities across the depletion layer by using the equilibrium expression (29.3) for the variation of the carrier densities in a potential $\phi(x)$. We have noted above that this expression follows from the fact that in equilibrium the drift currents are equal and opposite to the diffusion currents. In the general nonequilibrium case (e.g., in the diffusion region) the drift and diffusion currents are not in balance, and Eq. (29.3) does not hold. However, in the depletion layer there is a near balance between drift and diffusion currents,²³ and therefore to a reasonable approximation the carrier densities do obey Eq. (29.3), changing by a factor $e^{-e\Delta\phi/k_B T}$ as the depletion layer is crossed:

$$\begin{aligned} n_c(-d_p) &= n_c(d_n)e^{-e\Delta\phi/k_B T} = [n_c(d_n)e^{-e(\Delta\phi)_0/k_B T}]e^{eV/k_B T}, \\ p_v(d_n) &= p_v(-d_p)e^{-e\Delta\phi/k_B T} = [p_v(-d_p)e^{-e(\Delta\phi)_0/k_B T}]e^{eV/k_B T}. \end{aligned} \quad (29.48)$$

When $eV \ll E_g$, then V will be small compared with $(\Delta\phi)_0$, and the carrier densities on the minority side $[n_c(-d_p)$ and $p_v(d_n)]$ will continue to be very small compared with their values of the majority side $[n_c(d_n)$ and $p_v(-d_p)]$, just as they are when $V = 0$. Consequently, the conditions that the space charge vanish at the boundaries of the depletion layer,

$$\begin{aligned} n_c(d_n) - p_v(d_n) &= N_d, \\ p_v(-d_p) - n_c(-d_p) &= N_a \end{aligned} \quad (29.49)$$

give values for the majority carrier densities $n_c(d_n)$ and $p_v(-d_p)$ that differ from their equilibrium values N_d and N_a only by factors that are very close to unity. Thus, to an excellent approximation, when $eV \ll E_g$,

$$\begin{aligned} n_c(-d_p) &= [N_d e^{-e(\Delta\phi)_0/k_B T}]e^{eV/k_B T}, \\ p_v(d_n) &= [N_a e^{-e(\Delta\phi)_0/k_B T}]e^{eV/k_B T}, \end{aligned} \quad (29.50)$$

or, equivalently,²⁴

$$\begin{aligned} n_c(-d_p) &= \frac{n_i^2}{N_a} e^{eV/k_B T}, \\ p_v(d_n) &= \frac{n_i^2}{N_d} e^{eV/k_B T}. \end{aligned} \quad (29.51)$$

Substituting these results into the expression (29.47) for the total current, we find that

$$j = en_i^2 \left(\frac{D_n}{L_n N_a} + \frac{D_p}{L_p N_d} \right) (e^{eV/k_B T} - 1). \quad (29.52)$$

²³ This is verified in Problem 4.

²⁴ This follows from the form (29.6) of $(\Delta\phi)_0$ and the form (28.19) of n_i . It also follows directly from (29.50) by requiring that it yield the correct equilibrium values $n_c(-d_p) = n_i^2/N_a$ and $p_v(d_n) = n_i^2/N_d$, when $V = 0$.

This has the form (29.26) with the generation currents given explicitly by

$$\begin{aligned} J_e^{\text{gen}} &= \left(\frac{n_i^2}{N_a} \right) \frac{D_n}{L_n}, \\ J_h^{\text{gen}} &= \left(\frac{n_i^2}{N_d} \right) \frac{D_p}{L_p}. \end{aligned} \quad (29.53)$$

If we eliminate the diffusion constants appearing in (29.53) with the aid of (29.37), the expressions for the generation currents agree with the rough estimates (29.40) and (29.41).

PROBLEMS

1. The Depletion Layer in Thermal Equilibrium

(a) Show that if the exact (nondegenerate) form (29.3) is retained for the carrier densities, then Poisson's equation (which we approximated by Eq. (29.13) in the text) becomes the following differential equation for the variable $\psi = (e\phi + \mu - \mu_i)/k_B T$:

$$\frac{d^2\psi}{dx^2} = K^2 \left(\sinh \psi - \frac{\Delta N(x)}{2n_i} \right), \quad (29.54)$$

where $K^2 = 8\pi n_i e^2 / k_B T \epsilon$, $\Delta N(x)$ is the doping profile, $\Delta N(x) = N_d(x) - N_a(x)$, and n_i and μ_i are the carrier density and chemical potential for an impurity-free sample at the same temperature.

(b) The text discussed the case of a p - n junction made of highly extrinsic material, with $N_d, N_a \gg n_i$. In the opposite case of a lightly doped nearly intrinsic semiconductor, with

$$n_i \gg N_d, N_a, \quad (29.55)$$

we can find the electrostatic potential to high precision for an arbitrary doping profile, as follows:

(i) Assume that $\psi \ll 1$, so that $\sinh \psi \approx \psi$. Show that the solution to (29.54) is then given by:

$$\psi(x) = \frac{1}{2} K \int_{-\infty}^{\infty} dx' e^{-K|x-x'|} \frac{\Delta N(x')}{2n_i}. \quad (29.56)$$

(ii) Show that this solution and (29.55) imply that ψ is indeed much less than 1, justifying the initial ansatz.

(iii) Show that if ΔN varies along more than one dimension, then in the nearly intrinsic case:

$$\phi(\mathbf{r}) = e \int d\mathbf{r}' \Delta N(\mathbf{r}') \frac{e^{-K|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}. \quad (29.57)$$

(iv) The above result is identical in form to the screened Thomas-Fermi potential produced by impurities in a metal [Eq. (17.54)]. Show that the Thomas-Fermi wave vector [Eq. (17.50)] for a free electron gas has precisely the form of K , except that v_F must be replaced by the thermal velocity appropriate to Boltzmann statistics, and the carrier density must be taken as $2n_i$. (Why this last factor of 2?) The quantity K is the screening length of Debye-Hückel theory.

(c) Some insight into the general solution to (29.54) may be gained by the simple expedient of changing the names of the variables:

$$\psi \rightarrow u, \quad x \rightarrow t, \quad K^2 \rightarrow \frac{1}{m}. \quad (29.58)$$

The equation then describes the displacement u of a particle of mass m moving under the influence of a force that depends upon position (u) and time (t). In the case of an abrupt junction this force is time-independent before and after $t = 0$. Sketch the "potential energy" before and after $t = 0$, and deduce from your sketch a qualitative argument that the solution to (29.54) that becomes asymptotically constant as $x \rightarrow \pm\infty$ can vary appreciably only in the neighborhood of $x = 0$.

(d) Show that conservation of "energy" before and after $t = 0$ in the mechanical model for the abrupt junction described above permits one to show that the exact potential at $x = 0$ is that given by the approximate solution (29.14) plus a correction $\Delta\phi$, given by:

$$\Delta\phi = -\frac{kT}{e} \left(\frac{\sqrt{N_d^2 + 4n_i^2} - \sqrt{N_a^2 + 4n_i^2}}{N_d + N_a} \right). \quad (29.59)$$

Comment on how important a correction to ϕ this is, and how reliable the carrier densities (29.12) given by the approximate solution (29.14) are likely to be in the depletion region.

(e) As in (d), find and discuss the approximate and exact electric fields at $x = 0$.

2. Derivation of the Einstein Relations from Kinetic Theory

Show that the phenomenological equations (29.27) relating the carrier currents to the electric field and carrier density gradients, follow from elementary kinetic arguments such as were used in Chapter 1, with mobilities of the form (29.28) and (29.29), and diffusion constants of the form

$$D = \frac{1}{3} \langle v^2 \rangle \tau^{\text{coll}}. \quad (29.60)$$

Show that the Einstein relations (29.30) are satisfied provided the mean square thermal velocity $\langle v^2 \rangle$ is given by Maxwell-Boltzmann statistics.

3. Einstein Relations in the Degenerate Case

When dealing with degenerate inhomogeneous semiconductors one must generalize the equilibrium carrier densities (29.3) to

$$\begin{aligned} n_c(x) &= n_c^0(\mu + e\phi(x)), \\ p_v(x) &= p_v^0(\mu + e\phi(x)), \end{aligned} \quad (29.61)$$

where $n_c^0(\mu)$, and $p_v^0(\mu)$ are the carrier densities of the homogeneous semiconductor as a function of chemical potential.²⁵

(a) Show that the expression (29.9) for $\Delta\phi$ and the interpretation that precedes it continue to follow directly from (29.61).

(b) Show by a slight generalization of the argument on page 602 that

$$\mu_n = eD_n \frac{1}{n} \frac{\partial n}{\partial \mu}, \quad \mu_p = -eD_p \frac{1}{p} \frac{\partial p}{\partial \mu}. \quad (29.62)$$

²⁵ Note that the functional forms of $n_c^0(\mu)$ and $p_v^0(\mu)$ do not depend on the doping (though of course the value of μ does).

(c) In an inhomogeneous semiconductor, not in equilibrium, with carrier densities $n_c(x)$ and $p_c(x)$, one sometimes defines electron and hole *quasichemical potentials*²⁶ $\tilde{\mu}_e(x)$ and $\tilde{\mu}_h(x)$ by requiring the carrier densities to have the equilibrium form (29.61):

$$n_c(x) = n_c^0(\tilde{\mu}_e(x) + e\phi(x)), \quad p_c(x) = p_v^0(\tilde{\mu}_h(x) + e\phi(x)). \quad (29.63)$$

Show that, as a consequence of the Einstein relations (29.62), the total drift plus diffusion currents are just

$$\begin{aligned} J_e &= -\mu_n n_c \frac{d}{dx} \frac{1}{e} \tilde{\mu}_e(x), \\ J_h &= \mu_p p_v \frac{d}{dx} \frac{1}{e} \tilde{\mu}_h(x). \end{aligned} \quad (29.64)$$

Note that these have the form of pure drift currents in an electrostatic potential $\phi = (-1/e)\tilde{\mu}$.

4. Drift and Diffusion Currents in the Depletion Layer

Noting that the electric field in the depletion layer is of order $\Delta\phi/d$, $d = d_n + d_p$, and that the carrier densities there exceed their minority values substantially (except at the edges of the layer), show that the assumption that the drift (and hence diffusion) currents in the depletion layer greatly exceed the total current is very well satisfied.

5. Fields in the Diffusion Region

Verify the assumption that the potential ϕ undergoes negligible variation in the diffusion region, by estimating its change across the diffusion region as follows:

(a) Find the electron drift current at d_n by noting that the total electron current is continuous across the depletion layer and calculating explicitly the electron diffusion current at d_n .

(b) Noting that the electron density is very close to N_d at d_n , find an expression for the electric field at d_n necessary to produce the drift current calculated in (a).

(c) Assuming that the field found in (b) sets the scale for the electric field in the diffusion region, show that the change in ϕ across the diffusion region is of order $(k_B T/e)(n_i/N_d)^2$.

(d) Why is this indeed negligible?

6. Saturation Current

Estimate the size of the saturation electric current in a p - n junction at room temperature, if the band gap is 0.5 eV, the donor (or acceptor) concentrations $10^{18}/\text{cm}^3$, the recombination times 10^{-5} second, and the diffusion lengths 10^{-4} cm.

²⁶ Since we are not in equilibrium, $\tilde{\mu}_e$ need not equal $\tilde{\mu}_h$.